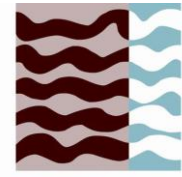




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Universidad Politécnica de Cartagena  
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# Ejercicio 1

## Cálculo de estructuras

Cartagena 2015

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Jorge Cerezo Martínez

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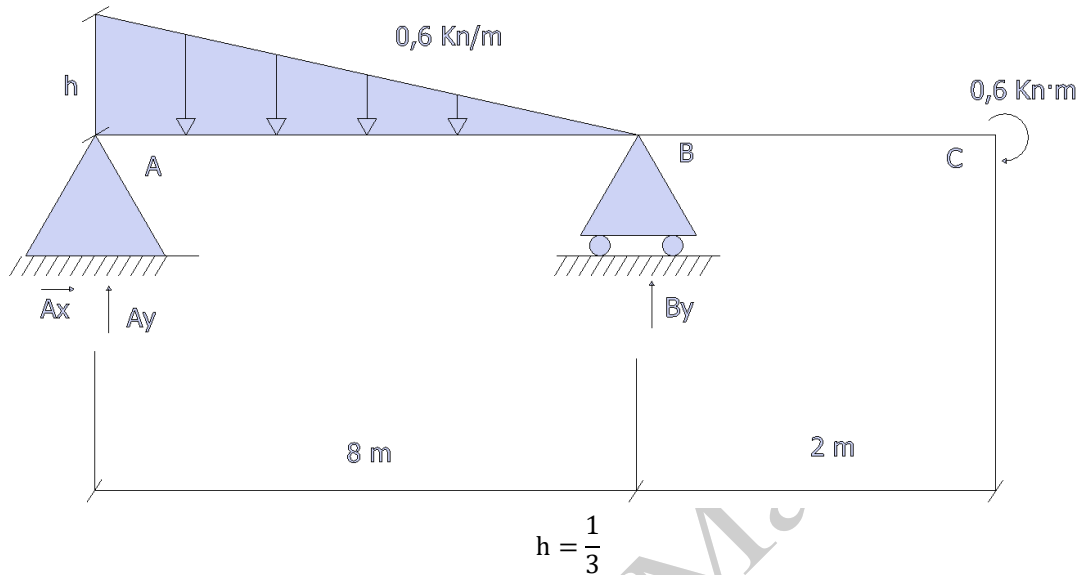
Ref. CA. 2.06

### Historial del documento

Fecha	Descripción	Rtdo.	Rvdo.	Apdo.
04/10/2018	Compilación de archivos	JCM	JCM	JCM

yhnt

Jorge Cerezo Martínez



Ecuaciones de equilibrio

$$\sum F_x = 0 \rightarrow A_x = 0$$

$$\sum F_y = 0 \rightarrow A_y + B_y - q \cdot L = 0 \rightarrow A_y + B_y - 0,6 \cdot \frac{8}{2} = 0 \rightarrow A_y = 2,4 - B_y \rightarrow A_y = 2,4 - 0,725 \rightarrow A_y = 1,675$$

$$\sum M_A = 0 \rightarrow 0,6 + 8 \cdot B_y + q \cdot \text{dist} \cdot (b \cdot a) \rightarrow 0,6 + 8B_y - 0,6 \cdot \frac{8}{2} \cdot \left(8 \cdot \frac{1}{3}\right) \rightarrow 0,6 + 8B_y - 6,4 = 0 \rightarrow B_y = \frac{6,4 - 0,6}{8} \rightarrow 0,725$$

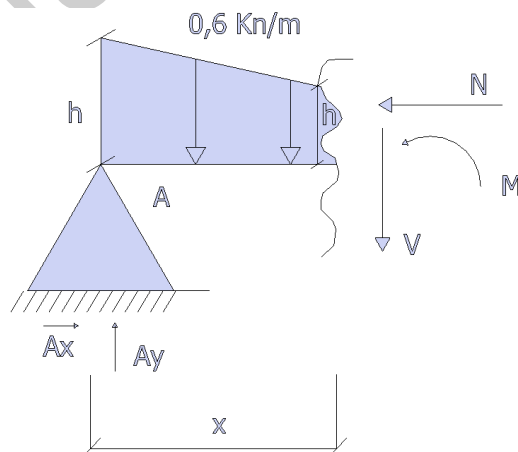
$$A_x = 0$$

$$A_y = 1,675$$

$$B_y = 0,725$$

Leyes de esfuerzo

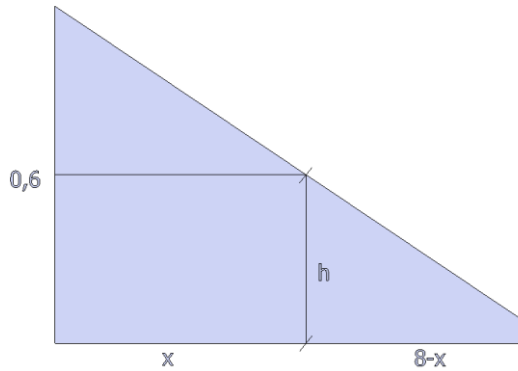
1<sup>er</sup> Corte



$$0 < x < 8$$

$$\sum F_x = 0 \rightarrow A_x + N = 0 \rightarrow N = 0$$

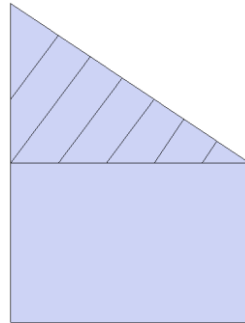
Triángulo



Relación de Tales

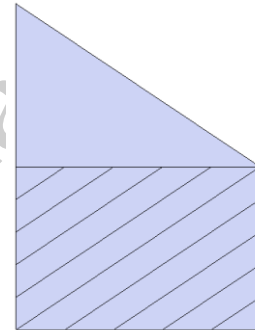
$$\frac{0,6}{8} = \frac{h}{8-x} \rightarrow h = \frac{0,6(8-x)}{8}$$

Triángulo, figura 1



$$R_1 = \frac{b \cdot a}{2} \rightarrow \frac{(0,6-h)x}{2} \rightarrow \frac{\left(0,6 - \frac{0,6(8-x)}{8}\right)x}{2}$$

Rectángulo, figura 2



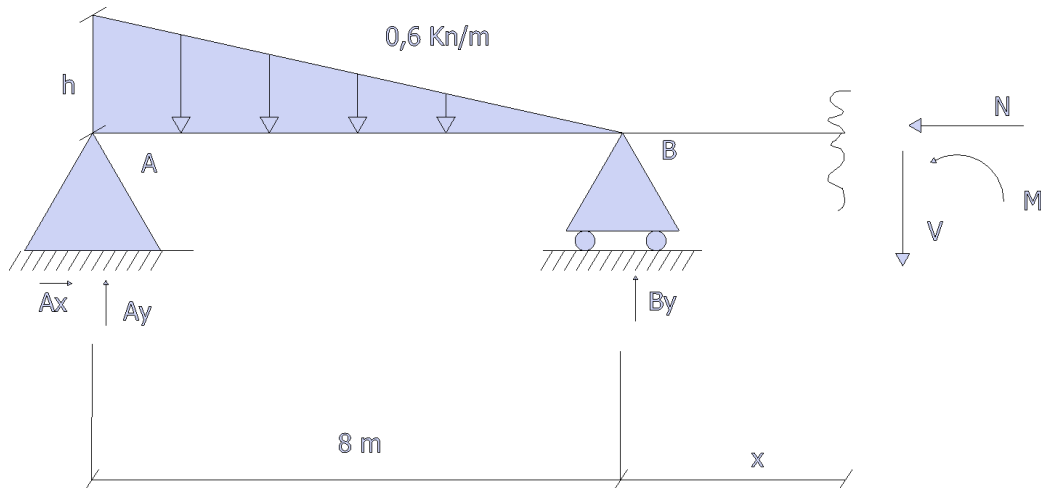
$$R_2 = h \cdot x = \frac{0,6(8-x)}{8}x$$

$$\begin{aligned} \sum F_y = 0 \rightarrow A_y - R_1 - R_2 - v = 0 \rightarrow v = A_y - R_1 - R_2 \rightarrow \\ \rightarrow \frac{\left(0,6 - \frac{0,6(8-x)}{8}\right)x}{2} - \frac{0,6(8-x)}{8}x + 1,675 \rightarrow -\left(0,6 - 0,6 + \frac{0,075x^2}{2}\right)x - (0,6x - 0,075x^2) + 1,675 \rightarrow \\ \rightarrow -0,0375x^2 + 0,075x^2 - 0,6x + 1,675 \rightarrow v = 1,675 - 0,6x + 0,0375x^2 \end{aligned}$$

$$\begin{aligned} \sum M = xA_y - \frac{3}{2}x \cdot R_1 - \frac{x}{2}R_2 - M \rightarrow M = 1,675x - \frac{3}{2}x \frac{\left(0,6 - \frac{0,6(8-x)}{8}\right)x}{2} - \frac{x}{2} \frac{0,6(8-x)}{8}x \rightarrow \\ \rightarrow -0,2x^2 + 0,2x^2 - 0,025x^3 - 0,3x^2 + 0,0357x^3 + 1,675x \rightarrow M = 1,675x - 0,3x^2 + 0,0125x^3 \end{aligned}$$

2° Corte

$8 < x < 10$



$$\sum F_x = 0 \rightarrow A_x + N = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow A_y + B_y - v - R = 0 \rightarrow 1,675 + 0,725 - v - 2,4 \rightarrow v = 0$$

$$\sum M = xA_y + (x-8)B_y - R\left(\frac{2}{3} \cdot 8 + (x-8)\right) - M = 0 \rightarrow 1,675x + 0,725x - 5,8 - 12,8 - 2,4x + 19,2 \rightarrow$$

$$\rightarrow M = 19,2 - 18,6 \rightarrow M = 0,6$$

Diagrama de esfuerzos

1er Corte

$0 < x < 8$

$$N = 0$$

$$V = 1,675 - 0,6x + 0,0375x^2 \begin{cases} x = 0; V = 1,675 \text{ Extremo} \\ x = 8; V = -0,725 \text{ Extremo} \\ x = 3,6; V = 0 \text{ Pto de corte} \end{cases}$$

$$\frac{-(-0,6) \pm \sqrt{(-0,6)^2 - 4 \cdot 1,675 \cdot 0,0375}}{2 \cdot 0,0375} \rightarrow \begin{cases} \frac{0,6 + 0,3297}{0,075} = 12,39 \text{ supera el rango} \\ \frac{0,6 - 0,3297}{0,075} = 3,6 \end{cases}$$

$$M = 1,675x - 0,3x^2 + 0,0125x^3 \begin{cases} x = 0; M = 0 \\ x = 8; M = 0 \end{cases}$$

$$0 = 1,675 - 0,3x + 0,0125x^2 \rightarrow \frac{-(-0,3) \pm \sqrt{(-0,3)^2 - 4 \cdot 0,0125 \cdot 1,675}}{0,0125 \cdot 2} \rightarrow \frac{0,3 \pm 0,79}{0,025} \rightarrow$$

$$\rightarrow \begin{cases} 43,6 \text{ supera el rango} \\ -19,6 \text{ valor negativo} \end{cases}$$

2° Corte

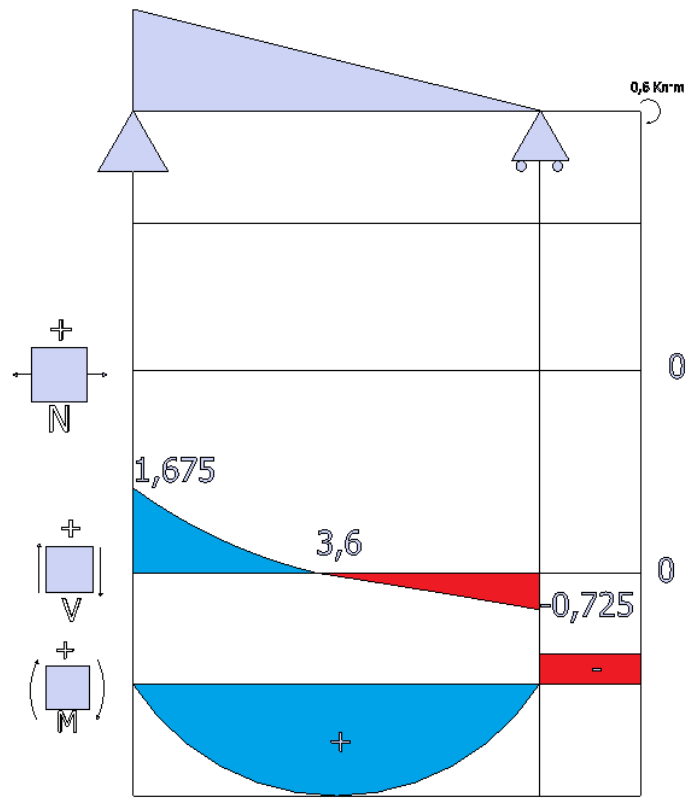
$8 < x < 10$

$N = 0$

$V = 0$

$M = -0,6$

Representación del diagrama



Carga ficticia



Ecuaciones del equilibrio

$$\sum F_x = 0 \rightarrow A_x = 0$$

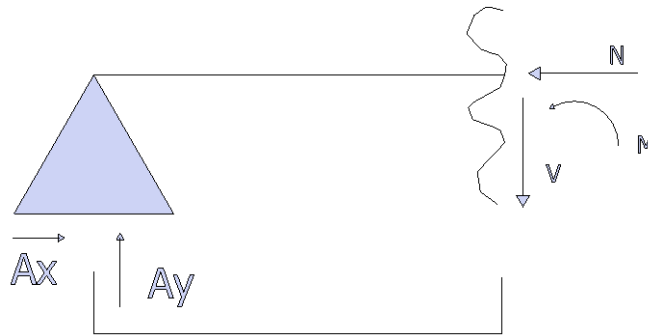
$$\sum F_y = 0 \rightarrow A_y + B_y - 1 = 0 \rightarrow B_y = 1 - A_y$$

$$\sum M = 0 \rightarrow 1 \cdot 10 + B_y \cdot 8 \rightarrow B_y = 1,25$$

$$A_y = 0,25$$

Leyes de esfuerzo

1<sup>er</sup> Corte

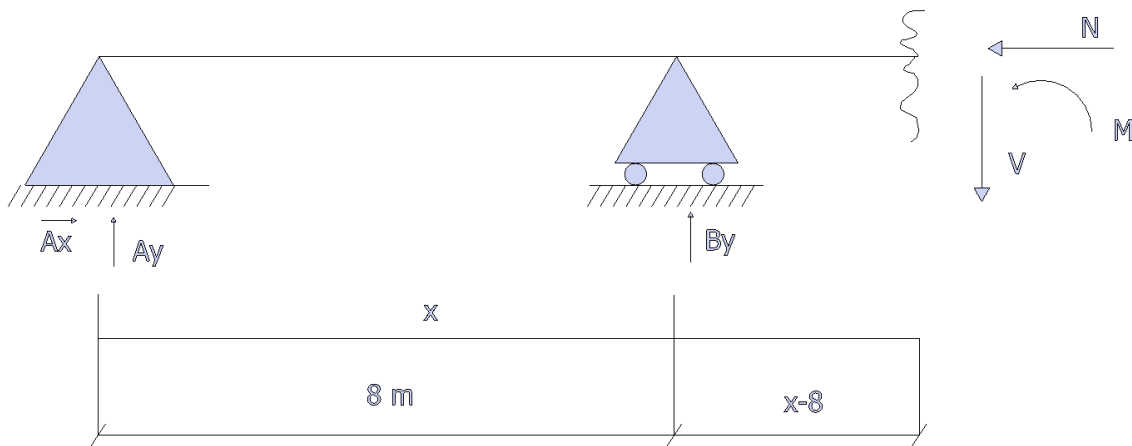


$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow A_y - v = 0 \rightarrow v = -0,25$$

$$\sum M = 0 \rightarrow A_y x - M \rightarrow M = -0,25x$$

2<sup>o</sup> Corte



$$\sum F_x = 0 \rightarrow N = 0$$

$$\sum F_y = 0 \rightarrow A_y + B_y - v = 0 \rightarrow v = 1,25 + 0,5 \rightarrow v = 1,5$$

$$\sum M = 0 \rightarrow A_y x + B_y(x - 8) - M \rightarrow M = 0,25x + 1,25x - 10 \rightarrow M = 1,5x - 10$$

$$0 < x < 8 \rightarrow M^* = -0,25x$$

$$8 < x < 10 \rightarrow M^* = 1,5x - 10$$

Desplazamiento

$$\delta_3 = \frac{1}{EI} \int M^* M dx \rightarrow \frac{1}{EI} \int_0^8 -0,25x \cdot (1,675x - 0,3x^2 + 0,0125x^3) dx + \frac{1}{EI} \int_8^{10} (1,5x - 10) (-0,6) dx \rightarrow$$

$$\rightarrow -\frac{1}{EI} \int_0^8 (0,4187x - 0,075x^2 + 0,05x^3) dx - \frac{1}{EI} \int_8^{10} (0,9x - 6) dx \rightarrow$$

$$\rightarrow -\frac{1}{EI} \left[ \frac{0,4287x^2}{2} - \frac{0,075x^3}{3} + \frac{0,05x^4}{4} \right]_0^8 - \frac{1}{EI} \left[ \frac{0,9x^2}{2} - 6x \right]_8^{10} \rightarrow -\frac{1}{EI} (51,7984 + 10,8) \rightarrow -\frac{1}{EI} 62,6$$